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Polynomial Ring

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Polynomial Over a Ring

An expression of the form

$$a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

is called a Polynomial, where the coefficients $a_0, a_1, a_2, \dots, a_n$ are numbers only.

Polynomial over a Ring

Let R be a ring. Let $a_0, a_1, a_2, \dots, a_n \in R$ be arbitrary

A polynomial over a ring R is defined as an infinite ordered system $(a_0, a_1, a_2, \dots, a_i, \dots)$ with at most a finite number of non-zero elements

Equal Polynomial — Two polynomials $(a_0, a_1, a_2, \dots, a_i, \dots)$ and $(b_0, b_1, \dots, b_i, \dots)$ are said to be equal if $a_i = b_i \forall i$.

A Polynomial $(a_0, a_1, a_2, \dots, a_i, \dots)$ is usually denoted by the symbol

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_ix^i + \dots$$

In this expression x is an arbitrary not belonging to R which commutes with every elements of R . Also $f(x)$ is called a polynomial in one indeterminate over the ring R .

a_0, a_1x, a_2x^2, \dots are called terms of this polynomial where a_0, a_1, a_2, \dots are called coefficients of the terms.